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Development Summary of a Sympathetic Discharge CO₂ Laser for LIDAR Use

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**Final Report
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A handwritten signature in black ink, appearing to read 'R. Barry Johnson', is positioned above a horizontal line.

**R. Barry Johnson
Principal Investigator**

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**Development Summary of a Sympathetic Discharge CO₂
Laser for LIDAR Use**

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A commercial pulsed sympathetic discharge laser has been characterized and modified for use as a potential lidar. This report summarizes the initial findings and modifications made to the baseline system. The new laser performance is then checked with theory and operational results are presented. The laser has inherent mode instability and high chirp. Several solutions were tried and their results are presented.

1 INTRODUCTION

This final report is meant to be used in conjunction with the papers and interim reports listed in the references section. This report summarizes the data found in those sources and where appropriate exactly reproduces portions of it. However, figures and pictorial information found in those documents are not reproduced here. This text does present a detailed mathematical model and analysis of the LP-140 absent in the other listed sources.

The Pulse Systems Inc. Model LP-140 is a commercial laser designed for materials marking and processing applications. The

laser is an attractive candidate for a lidar because of the relatively high energy pulses and repetition rates achievable. The configuration built for MSFC is capable of delivering over 1 J at repetition rates up to 7 Hz.

This laser also has inherent weaknesses in operation that limit its direct use as a lidar. For example, although the pulse energies are high, the pulse lengths tend to be greater than 15 μ s, and the laser exhibits extreme spatial mode instability.

The goal of this development effort was to see if the LP-140 could be economically modified without complete redesign to improve its operational characteristics. Priority was placed on maintaining the general LP-140 physical appearance and limiting the physical design to inexpensive and simple modifications that are readily reproducible by an existing owner of an LP-140 without requiring engineering redesign. This final report summarizes this development effort.

2 LASER DESCRIPTION

The LP-140 built for NASA and used during this investigation differs somewhat from the standard commercial LP-140. Firstly, the active region was extended from the original 1.7 L volume to 2.2 L. The standard gas mixture of CO₂:N₂:CO:He, 18:15:2:65 which normally produces pulses in excess of 15 μ s in length was changed by PSI after utilizing a proprietary design code to 15.29:17.33:1.99:65.39 respectively.[1]

The NASA LP-140 included a a Burleigh Instruments Model I-1000 reflection grating mounted at the Littrow angle for single longitudinal mode operation. Each gain arm has an effective volume of $4 \times 4 \times 70 \text{ cm}^3$, comprised of four equal discharge volumes, with an unsaturated gain of 0.031 cm^{-1} and a saturation intensity of 0.017 J cm^{-2} .

The standard gas handling system was re-engineered by PSI to extend the commercial operational lifetime and minimize downtime. The commercial housing was maintained but accommodations were made for the grating. The mechanical resonator remained a standard single cast aluminum structure but of slightly longer length than normal.

The laser was secured with a Wallace Tiernan model FA 160 gauge in place and PSI indicated that the laser should be operated between 30 - 50 torr as read at this pressure gauge.

The following experiments were performed using this pressure gauge and no attempt was made to determine the actual pressures within the laser volume itself. At best this gauge then becomes a relative indicator and baseline for the following experiments. Also, since the laser is designed to be operated in a gas flowing configuration, the pressure drop across the gauge is also dependent on this gas flow as well as the gas mixture.

2.1 Optical description

The NASA LP-140 folded laser cavity is comprised of 5 elements. The first element is a 6.35 cm diameter x 0.63 cm edge thickness meniscus ZnSe output coupler with a 5m radius of curvature on both surfaces. The element is only A.R. Coated on the convex side which also faces into the cavity to provide magnification to the incident wavefront. The center of this element has a square gold mirror flashed on the surface with an area of 4 cm². Next two rectangular flat mirrors fold the optical path through the second arm. Both of these mirrors are first surface gold coated. PSI informs us that they are $\lambda/4$ at 632.8 nm or better. These mirrors are secured by simple analog three-point mounts and secured by epoxy. The laser tube cavity is sealed by the fourth optical element. This is a ZnSe plano-convex lens with a 10 m focal length. Both surfaces are A.R. Coated and the convex side faces the discharge. The optical cavity is completed by the fifth element which is the model I-1000 reflection grating 10 cm beyond the final lens element #4. The total optical cavity length is 238 cm. Because the 4 x 4 cm² discharge region is square all optical elements are underfilled. The actual beam dimensions emerging from the output coupler are 4 x 4 cm² with the center obscured by a square.[1]

2.2 Electrical description

The simple sympathetic discharge circuit is duplicated for all eight discharge electrode pairs within the LP-140 volume. The only exception is the Thyratron and $880\text{ M}\Omega$ bypass resistor which are the same in all the circuits. The primary discharge circuit consists of a $0.1\text{ }\mu\text{F}$ capacitor and the second is made up of three 22 nF capacitors. Because of the inductor in series with the $0.1\text{ }\mu\text{F}$ capacitor, the circuit has an inherently high time constant in relationship to the 22 nF circuit. The 22 nF circuit creates a minimal energy but fast risetime voltage pulse across the laser head when the thyratron is activated. The initial pulse represents the preionisation between the grid and pin electrodes. After a finite delay the slower main discharge pulse arrives at the laser head, the main discharge gap is preionized and a uniform breakdown in the gas follows. Energy is prevented from being deposited into the preionisation discharge by the slower main pulse by the small 500 pF capacitor.[4]

3 THEORETICAL MODEL

The existing model for the LP-140 is a general simplification of the LP-140 resonator to the gain cell case. PSI stated that this approach proved useful and sufficient in scope to form the engineering baseline for the NASA LP-140, as such the theory as dictated by PSI is summarized here and their model is outlined for

easy reference.

3.1 Existing model

PSI has indicated that a reasonable approximation for the prediction of output power from the NASA LP-140 laser can be obtained by taking into account the change rate of the excited number densities for the N_2 and CO_2 in the active gain volume of the laser. The model is simplified to include the output power relationship to the product of the input power signal times the total gain (G) in a linear fashion. This model does not take into account saturation effects or nonlinear transients such as LIMP. PSI sets up the relationship as:[7]

$$P_{out}(t) = P_e(t)G(t) \quad 1.0$$

This model does not allow for a wavefront build up from noise but requires a signal to be injected into the gain volume, consequently the input power, $P_e(t)$, represents the pulse to be amplified. The input pulse shape is assumed to be gaussian in spatial distribution and with a square time dependence. The gaussian shape is supposed to fall to the 1/e point at a radius of $r = 2$ cm. For modeling purposes such pulse can be described as a simple Fourier series:[14]

$$f(x) = \frac{4a}{\pi} \left[\frac{\cos b}{1} \sin x + \frac{\cos 3b}{3} \sin 3x + \frac{\cos 5b}{5} \sin 5x + \dots \right]. \quad 1.1$$

The gain function $G(t)$, is assumed to be

$$G(t) = e^{A(t)L} \quad 1.2$$

$A(t)$ is defined as the time dependent small signal gain coefficient while L is the total active gain length of the laser volume. We define

$$A(t) = k_A N_{CO_2}(t) \quad 1.3$$

Here the constant k_A can be approximated from theory or measured directly. PSI opted for the latter and reported a small signal gain for the LP-140 of $G_o = 0.032/\text{cm}$. Laboratory measurements made at NASA confirm this with a measured reading of 0.031cm^{-1} . In practice the small signal gain varies with time as the laser medium is perturbed, however, the model does not account for this and assumes instead that the gain remains constant over time. The value of $N_{CO_2}(t)$ is then reduced to a constant representing the number density of the excited CO_2 molecules at the time of peak. If this number density is defined as $N_{CO_2}(G_o)$, a pressure dependence must also be considered for k_A for use throughout a range of operational pressures. The model simplifies this relationship and assumes that the value $N_{CO_2}(G_o)$ increases in direct proportion to the

pressure described as [15]

$$k_A = \frac{G_o}{[N_{CO_2}(G_o) \left(\frac{N_{MOL}(P)}{N_{MOL}(30)} \right)]} \quad 1.3$$

$N_{MOL}(P)$ defines the number density of the CO_2 molecules available for at a given pressure P and $N_{MOL}(30)$ represents the number density at the operational pressure of 30 torr, this of course may vary and in practice was found to be optimal at the higher pressure end (>50 torr). The constant cross section k_A is simply the excitation cross-section for the CO_2 molecules. Equation 1.3 becomes time independent but pressure dependent for the cross-section term.

The PSI model allows the density of excited CO_2 molecules to be dependent on three factors, these are the excitation rate by the N_2 molecules $D_{NCO_2}(t)$, the spontaneous emission rate to lower CO_2 states $D_{NCO_2SE}(t)$ and the rate of energy depletion from the laser discharge by virtue of the amplification of the incident signal $P_{out}(t)$. PSI models the time dependent number density per cubic centimeter as [7, 14, 15]

$$N_{CO_2}(t + \delta t) = N_{CO_2}(t) + D_{NCO_2}(t) - D_{NCO_2SE}(t) - P_{out}(t) \quad 1.4$$

Since the principle excitation term for the CO₂ molecules is dependent on the number density of the N₂ molecules, N_{N2}(t), the resulting partial pressure of the CO₂, P(CO₂), and the rate coefficient, K_{nc}, are all coupled for the transfer. Therby the excitation term can be modeled by:

$$D_{NCO_2}(t) = NubN_2(t)P(CO_2)e^{(K_{nc}\delta t)} \quad 1.5$$

The theoretical rate coefficient used by the PSI model is 19608/ sec. Because of the coupled nature of the number density terms of the excited molecules and the partial pressures, the resulting equation for spontaneous emission used by PSI becomes:

$$D_{NCO_2SE}(t) = N_{CO_2}(t)P(CO_2)e^{(-K_{cc}\delta t)} \quad 1.6$$

Here the rate coefficient term for spontaneous emission used by PSI is 1050/sec.

Since the energy lost during energy extraction by the amplifier medium is dependent on the gain function, G(t) and the total number density of molecules subject to relaxation, the relation is modeled as

$$\frac{G(t) \epsilon_{sun} \epsilon(t)}{[h \nu(CO_2) volume]} \quad 1.7$$

where h is Planck's constant, $\nu(CO_2)$ is the laser frequency which at $10.6 \mu m$ becomes $(2.833 \times 10^{13} \text{ Hz})$ and volume is the total active discharge volume while $DE_{IN}(t)$ is the energy input signal as a function of time. [7, 14, 15]

$$\epsilon_{\epsilon}(t) = P_{\epsilon}(t) \delta t \quad 1.8$$

Here the value of $P_{IN}(t)$ is described by the form of the injected signal.

Because the relative number density of excited N_2 molecules is mostly dependent on losses due to collisional transfer with relaxed CO_2 molecules ($-D_{NCO_2}$), and gain due to electrical excitation with an input power function of $P_o(t)$. The N_2 excitation is modeled by PSI to take place instantaneously in comparison with the other transition rates in this model. The actual number of excited N_2 molecules per volume is dependent on the form of the electrical pumping power and the transfer efficiency from the pump electrons to the N_2 itself. This includes the energy absorbed by the N_2 when excited ($E_N = h \nu(N_2)$). The final expression for the PSI model becomes: [7]

$$N_{N_2}(t + \delta t) = \frac{N_{N_2}(t) - D_{NCO_2}(t) + P_o(t)\delta t E_{ff}}{(E_N volume)} \quad 1.9$$

The PSI model divides the gain region into a number of annular zones for which the gain is calculated separately. The model also assumes uniform excitation across the gain region. PSI limits the number of annular zones for their model to 5, but this figure remains variable.

Annular zones with uniform gain are truncated at radius values of $r = 2$ cm or greater. The input signal is injected into the cavity rather than being generated within it spontaneously. The electrical discharge pump can be modeled as either a single or a double exponential (exponential rise and fall time), or a pair of double exponential pulses. The waveforms are derived from: [14]

$$P = A(e^{-bt} - e^{-at}) \quad 2.0$$

For detailed modeling this pulse form is described as

$$P(t) = P_o \left[\text{rect} \left[\frac{t - \left(\frac{\tau}{2}\right)}{\tau} \right] + \text{rect} \left[\frac{t - T + \left(\frac{\tau}{2}\right)}{\tau} \right] \right] \quad 2.1$$

where P_0 is the average power level, τ is the duration of the individual pulses, and T is the total duration of the waveform.

The following table summarizes the variables and inputs allowed by the PSI model:[7]

PARAMETERS	Default	Normal Range
Pressure (torr)	30	50
CO2 pressure (torr)	6	variable
electrical pump energy (J/l)	10	25
peak input electrical power in k-Watts	618.89	calculated
small signal gain coeffi- cient (1/cm)	0.32	.030-.050
gain length in cm	238	variable
time duration for run	-	variable
time step in micro seconds	-	variable
input signal delay in micro- seconds	-	variable
injected signal power in watts	-	variable
rise time constant for elec- trical pulse in 1/micro- seconds	0.99	fixed

fall time constant for electrical pulses in 1/microseconds	0.06	fixed
conversion efficiency (EFO) =	0.2401	calculated
volume of gain region (liters)	-	calculated
number of annular zones	-	variable
aperture radius (cm)	1.5	2
1st stage transmission factor	-	calculated

The code used by PSI to model the Boltzmann distribution was developed by the Kirtland Air Force Weapons Lab and is available as share ware.[15]

3.2 Extended theory

3.2.1 LIMP

Laser induced medium perturbation is a nonlinear phenomena arising from the interaction of the self organizing laser wavefront and the active gain medium. The interaction of the laser mode with the medium can result in a frequency drift and discharge instability as well as frequency instability or chirp. If the instability is characterized by large frequency oscillations the far-field wavefront performance can be seriously degraded. Dr. Martin Smithers of MSFC has generated an outstanding diffraction code to model the anticipated output from the LP-140. The equations used in his model were presented in a short seminar at MSFC and he graciously provided me with a copy of his diagrams, since I am not aware of a publication containing them for sake of completeness I have elected to insert them here in their entirety as found in his view graphs. Dr. Smithers should be consulted for model clarification and further detailed explanations. The volumetric heating rate of the laser medium is defined as [10]

$$Q = \frac{v_2 g l_s}{V} + \frac{v_1 g l}{V} \quad 2.2$$

where $I_s = 2.5 \text{ kW/cm}^2$, the linearized hydrodynamic equation becomes

$$\frac{\partial}{\partial t} \left(\frac{\partial^2 \rho}{\partial t^2} - C_s^2 \nabla^2 \rho \right) = (\gamma - 1) \nabla^2 Q \quad 2.3$$

The Gladstone-Dale equation is defined as

$$\delta n = K_\rho \quad 2.4$$

Simple saturated gain is defined as

$$g = \frac{g_o}{\left(\frac{l+2l}{l_s} \right)} \quad 2.5$$

$$E' = E \exp(g L g) \quad 2.6$$

The one dimensional pressure perturbation is defined as

$$p(t) = -\frac{(\gamma-1)}{C_s^2} \int_0^t dt' (Q(x, t') - 1/2 Q(X + C_s(t-t'), t') - 1/2 Q(X - C_s(t-t'), t')) \quad 2.7$$

$$E' = E \exp \left(-i 2\pi K_\rho 2 \frac{Lg}{\lambda} \right) \quad 2.8$$

Eigenvalue Equation

$$E' = \gamma E \exp(-ik2L) \quad 2.9$$

Complex Eigenvalue

$$\gamma = |\gamma| \exp(i\phi) \quad 3.0$$

Frequency

$$f = \frac{c}{2L} \left(9 + \frac{\phi}{2\pi} \right) \quad 3.1$$

Frequency Shift

$$\delta f = \frac{c}{2L} \left(\frac{\delta \phi}{2\pi} \right) \quad 3.2$$

Unstable resonator with a gaussian reflectivity mirror,
where the reflectivity of the output mirror is defined as

$$R(x) = R_0 \exp\left(-\frac{X^2}{W_m^2}\right) \quad 3.3$$

Half Width of gaussian mode:

$$W_b^2 = (M^2 - 1) W_m^2 \quad 3.4$$

Outcoupling Ratio

$$\Gamma = 1 - \frac{R_0}{M^2} \quad 3.5$$

where the laser parameters are defined as follows:

GAIN MEDIUM [10]

$$\gamma = 1.47, K = 0.28 \text{ cm}^3 / \text{gm } g_0 = 0.15 / \text{cm}$$

$$I_s = 2.5 \text{ kW/cm}^2, L_g = 70.0 \text{ cm}, \lambda = 10.6 \mu m$$

$$C_s = 430 \text{ m/s}, \frac{v_2}{v} = 2.44, \frac{v_1}{v} = 1.47$$

UNSTABLE RESONATOR [10]

$$M = 2.0, N_{eg} = 1.375, L = 250 \text{ cm}, 2a_s = 1.707 \text{ cm}$$

GRM UNSTABLE RESONATOR

$R_o = 0.70$, $M = 1.67$, $L = 250$ cm, $a_o = 2.0$ cm, $w_b/a_o = 0.24$,
0.4, 0.7

Dr. Smithers was able to conclude that the use of a GRM unstable resonator delays the onset of instability and that a narrow beam is necessary to preserve a smooth gaussian profile. He has also determined that a possible loss in gain saturation may be offset by enhanced brightness in the far field. [10]

3.2.2 Lidar equations

As part of the development of the LP-140 as a possible lidar candidate it is necessary to gain insight into the theoretical performance and requirements imposed on the system by the physics of the requirements. The following sets of equations are an attempt at providing a simplified but concise lidar model showing the relationship of the source parameters to the transmission and reflection from backscatter. In the time allotted the model was not extended to account for the extended nature of B backscatter coefficients, rather this simple model assumes a fixed hard target

with a predictable reflectivity. The model lends itself well to being implemented on an HP-48SX hand held computer for simulation. The source code for this model is included in the appendix.

All transmitted waveforms of the same energy produce the same peak signal-to-noise ratio at the output of an appropriate matched filter. The detailed time structure of the waveform influences only the range and velocity resolution capabilities of the Lidar system. A heterodyne system must be able to detect a pulsed sinusoid envelope inside of narrowband gaussian noise. This is also the problem associated with range measurements alone. [12]

Simultaneous range and velocity measurements can be made by a pulsed energy detection system if the transmitted signal is power modulated in such a way that the doppler shift of the modulation waveform can be determined from the time variations of the detected photocurrent or voltage. The best power modulation waveform must be empirically derived.

The final selected waveform is more important in the energy detection case than in the heterodyne case because the structure influences both range and velocity resolution.

Velocity measurement is based on the observed Doppler compression or expansion of the total signal of duration T . In a simple coherent detection model the photodetector such as

an SAT liquid nitrogen cooled HgCdTe system is followed by a counter that counts for a time equal to a range-gate interval of duration τ , which is the width of the individual signal pulse within the pulse envelope. [12]

3.2.2.1 Upper performance determination assuming a quantum noise limited case

the probability is denoted by P_c , it is found from detection probabilities and is related by [12]

$$P_c(W) = \left[P_d\left(\frac{W}{z}\right) \right]^z \quad 3.6$$

where $P_c(W)$ = means the value of P_c for a target energy return of W , and $P_d(W/z)$ means the value of the detection probability for a target return energy W/z .

To calculate P_c for a total target return energy W , both of the receiver pulses of energy W/z must fire the threshold circuit. In a quantum noise limited case there is no possibility of false alarm. Thereby for low noise levels and for reasonably low probabilities of false alarm, the error probability of performance of the energy detection system can exceed the heterodyne lidar system.

[12]

3.2.2.2 Heterodyne range resolution

Heterodyne range resolution is defined as δR_H , which is: [12]

$$\delta R_H = \frac{c}{2B} \quad 3.7$$

here c = light speed or $\sim 3 \times 10^8$ m/s and B = the Bandwidth of the transmitted signal. The bandwidth can be approximated by

$$B = \frac{1}{T}, = B = \frac{1}{\tau} \quad 3.8$$

depending on which bandwidth is being considered.

3.2.2.3 Energy detection range resolution

Energy detection range resolution δR_E is defined as [12]

$$\delta R_E \cong \frac{c\tau}{2} \quad 3.9$$

where τ is the length of the individual pulse inside of a pulse envelope or train. This remains accurate for other waveforms as long as τ is understood to measure the fine structure of the transmitted wave pulse.

If waveforms of approximately equal complexity are used the parameters B^{-1} and τ are of the same order of magnitude such that

$$\frac{1}{B} \cong \tau \therefore B \cong \frac{1}{\tau} \quad 4.0$$

and the range resolution of the two systems are comparable.

3.2.2.4 Heterodyne frequency resolution

Heterodyne frequency resolution δf_H is defined as [12]

$$\delta f_H \cong \frac{1}{T} \quad 4.1$$

from this we get velocity resolution δV_H defined as

$$\delta V_H \cong \frac{c}{2\sqrt{T}} \quad 4.2$$

where ν is the optical frequency defined as $\nu = c/\lambda$ and T = duration of the entire transmitted waveform.

For the case of a simple double pulse waveform, targets with resolvable velocities introduce time compressions or time expansions of the total interval T that differ by a minimum of a single pulse interval τ . [12]

3.2.2.5 Energy detection velocity resolution

Energy detection velocity resolution δV_E is defined as [12]

$$\delta V_E \cong \frac{c\tau}{2T} \quad 4.3$$

this applies to other waveforms as long as the τ defines the fine structure.

3.2.2.6 comparison

Heterodyne velocity resolution exceeds energy detection by the factor [12]

$$\delta \frac{V_E}{\delta V_H} = \nu\tau \quad 4.3$$

In space applications where there is little or no backscatter, the energy detection system will outperform the heterodyne system. High velocity resolution however,

mandates the use of a heterodyne detection system, if wide ranges of frequencies can be adequately monitored and the receiver has a large bandwidth. For such a case digital filtering is almost a necessity. For the case of 1 microsecond long laser pulse a theoretical velocity resolution of 5.29 m/s is obtained, while the frequency resolution for the heterodyne case is 1×10^6 Hz. The bandwidth of the signal becomes 1×10^6 Hz and the range resolution becomes 150 m. The required electronic bandwidth is simply

$$\frac{2}{\pi}(B) \quad 4.4$$

or 1.57×10^6 Hz.

3.2.2.7 Detailed model

The following series of equations encompasses a simplified but functional lidar model.

The basic equation describing the power received at the detector P_R in (watts) is [11, 12]

$$P_R = \left(\frac{P_T}{R^2 \Omega_T} \right) (\delta A_R) \left(\frac{A_c}{R^2 \Omega_R} \right) T^2 \quad 4.5$$

where P_T is the transmitted power in watts, R is the range in meters, where this term is squared because of the inverse square relationship of signal strength over distance as $(1/R^2)$. Ω_T = the solid angle of the transmitted beam (SR) for which one degree = .0345 SR, δ = is ρ or common reflectivity of the target in percent where $\delta = \rho = < 1$, and A_R = is the area of the target in m^2 , A_c = clear aperture of the receiver (m^2), Ω_R = solid angle of the receive beam from the target (SR), and T = transmissivity of the optical path including the sum of losses and nonlinear contributions such as B backscatter, Rayleigh, and Mie scattering.

This model is very flexible and each of the 4 terms may be easily expanded to encompass more accurate models. Term #1 describes the transmitted beam, #2 the target characteristics, #3 the return beam, and #4 the optical path. As an example an improved form replaces the simple target description by one adopted by some of the teams developing a LAWS model such as [6, 10]

$$P_b(t) = \int_{c(t-\tau_r)/2}^{ct} P_T(t-2R/c) B(R) (A_{det}/R^2) \eta_h \eta_o O_R T(R) dR \quad 4.6$$

where $P_T(t-2R/c)$ = transmitted power, $B(R)$ = Backscatter coefficient, η_h = heterodyne mixing efficiency, η_o =

system optical efficiency, $T(R)$ = two way transmissivity,
 $O(R)$ = range dependent overlap, A_{det}/R^2 = distance
factor. For the anticipated power spectrum the probe beam
is divided into N subvolumes throughout the target volume
where the contribution of each sub volume is the
normalized power spectrum due to the superposition of all
subvolumes shown as

$$S(f_i) = \frac{N(f_{i-1} < f < f_i)}{N_{Tot}} \quad 4.7$$

$$S(f, R_i) \sim \sigma(f - f_D, i) \quad 4.8$$

$$f_{o,i} = \frac{2V_r(R_i)}{\lambda} \quad 4.9$$

$$S_{(f)} = \frac{N(f_{i-1} < f, f_i)}{N_{Tot}} \quad 5.0$$

The relational expressions are now presented in a tabular
form with the definitions of the variables following the
equation list. If these expressions are entered and
stored sequentially in the HP-48SX they may be used

directly as a functional model by supplying the known information and allowing the 48 to solve for the remaining unknowns.

[11, 6, 10, 4 , 8]

$$P_r = \left(\frac{P_t}{R^2(\Omega_r \times .034)} \right) (\rho A_r) \left(\frac{A_c}{R^2(\Omega_r \times 0.34)} \right) T^2 \Rightarrow (Watts) \quad 5.1$$

$$P_t = \frac{E_t}{t} \Rightarrow (watts) \quad 5.2$$

$$E_r = P_r t \Rightarrow (Joules) \quad 5.3$$

$$n_t = \frac{E_t}{\left(\frac{hc}{\lambda} \right)} \Rightarrow (photons) \quad 5.4$$

$$n_o = \frac{E_r}{\left(\frac{hc}{\lambda} \right)} \Rightarrow (photons) \quad 5.5$$

$$(u(t), v(t)) = \frac{\eta q}{hf_o} [P_{is} + P_r(t) + 2\sqrt{P_{is}P_r(t)} \cos(f_o - (f_o 2\pi)) + R + \phi_r(t)] \Rightarrow (I.V) \quad 5.6$$

$$f_o = \left(\frac{c}{\lambda} \right) 2\pi \Rightarrow (Hz) angular \quad 5.7$$

$$P_{lo} = \frac{P_t}{A_t} \Rightarrow (watts) \quad 5.8$$

$$P_s(t) = \frac{P_r}{Pol} \quad 5.9$$

$$f_b \cong \left(\frac{2V}{\lambda} \right) + f = \left(f' = \left(\frac{f \left(1 - \left(\frac{v}{c} \right) \right)}{\sqrt{\left(1 - \left(\frac{v}{c} \right)^2 \right)}} \right) \right) \Rightarrow (Hz) \quad 6.0$$

$$fringes) = \frac{2V}{\lambda} \Rightarrow (Hz) \quad 6.1$$

$$db \cong 10 \text{Log}_{10} \left[\frac{\eta P_r(t)}{hf} \right] = \left(db = 10 \text{Log}_{10} \left[\frac{\eta W(t)}{hf} \right] \right) \Rightarrow (S/N) \quad 6.2$$

$$W(t) = 2 \left[\frac{\eta^2 q^2}{hf^2} \right] P_{lo} \int_0^{t_R} P_s \sin(t) dt \Rightarrow (watts) \quad 6.3$$

$$\Delta(fringes) = \frac{2V \sin \alpha}{\lambda} \Rightarrow (Hz) \quad 6.4$$

$$t_R = t + \Delta t \Rightarrow (s) \quad 6.5$$

$$f = \frac{c}{\lambda} \Rightarrow (Hz) \quad 6.6$$

$$S/N \cong \left(\frac{\eta P_r}{h f} \right) = \left(S/N = \left(\frac{\eta W(t)}{h f} \right) \right) \Rightarrow (absolute) \quad 6.7$$

$$(axial - mode) = \frac{2\pi}{\left(\frac{2L}{c} \right)} \cong (W_1, W_2) \Rightarrow (Hz) \quad 6.8$$

$$I_{mt} = \frac{\left[I_1 \sin \left(N_1 \omega_1 \left(\frac{t_1}{a_1} \right) + \phi_1 \right) \right]^2}{\left[I_2 \sin \left(N_2 \omega_2 \left(\frac{t_1}{a_2} \right) + \phi_2 \right) \right]^2} \Rightarrow (\sin - envelope) \quad 6.9$$

$$(distance) = c \left(\frac{time}{2} \right) = (time - of - flight) \Rightarrow (m) \quad 7.0$$

$$\frac{Delta V_E}{\Delta V_H} = \frac{c \tau}{2 T_P} \Rightarrow (m/s) \quad 7.1$$

$$Delta V_H = \frac{c}{2 f T_P} \Rightarrow (m/s) \quad 7.2$$

$$Delta f_H = \frac{1}{T_P} \quad 7.3$$

$$BW = \frac{1}{\tau} \Rightarrow (Hz) \quad 7.4$$

$$\Delta R_E = \frac{c\tau}{2} \quad 7.5$$

$$\Delta R_H = \frac{c}{2BW} \quad 7.6$$

$$P_\tau = P_o \left[\text{rect} \left[\frac{t_1 - \frac{\tau}{2}}{\tau} \right] + \text{rect} \left[\frac{t_1 - T + \frac{\tau}{2}}{\tau} \right] \right] \Rightarrow (\text{Dbl-pulse}) \quad 7.7$$

$$R = \frac{(\text{time})}{c} \Rightarrow (m) \quad 7.8$$

$$\Delta R = \frac{c}{2BW} = \frac{\lambda}{2 \left(\frac{BW}{f} \right)} \Rightarrow (m) \quad 7.9$$

$$f_\epsilon = \frac{2}{\lambda} V_i \Rightarrow (Hz) \quad 8.0$$

$$\vec{V}_i = (\vec{\Omega} \times \vec{r}_i) \cdot \vec{R} \quad 8.1$$

$$\delta f_\epsilon = \frac{2\Omega_\Delta X_i}{\lambda} \quad 8.2$$

$$\Delta X_i = \frac{\lambda \delta f_\epsilon}{2\Omega} \quad 8.3$$

$$\delta f d \cong \frac{1}{T_m} \quad 8.4$$

$$\Delta X = \frac{\lambda}{2\Omega T_m} \quad 8.5$$

$$\Delta X_1 = \frac{\lambda}{2\Delta\phi} \quad 8.6$$

$$\Delta X_2 = \frac{\lambda \delta f}{2\Omega} = \frac{\lambda}{2} \frac{f}{\Omega_{ri}} = \frac{\lambda}{2} \frac{f}{\Delta\phi} \quad 8.7$$

$$\Delta R_1 = \frac{ct_p}{2} = \frac{c}{2BW} = \frac{\lambda}{2\left(\frac{BW}{f}\right)} \quad 8.8$$

$$(fdi) = \frac{2}{\lambda} (\vec{\Omega} \times \vec{f}_i) \cdot \frac{\vec{R}}{|\vec{R}|} \quad 8.9$$

$$T_a(t) = \Gamma \exp(-.001 \sigma R_{(t)}) + \frac{\sigma S}{4\sigma} [1 - \exp(-.001 \sigma R_{(t)})] \quad 9.0$$

$$Pb_t = .25[H_{IS}B_o + H_S X] \alpha_R^2 A_r T_r T_a(t) \Rightarrow (watts) \quad 9.1$$

$$P_{st} = \frac{P_t T_t \Gamma A_r T_r}{\pi R_{(t)}^2} \exp[-.002 \sigma R_{(t)}] \Rightarrow (watts) \quad 9.2$$

$$SNR_{a(t)} = 10 \log_{10} \left[\frac{B^2 P_{st}^2 R_L G^2}{2qB[\beta P_{bt} I_d] R_L G^2 + 2FkTB} \right] \quad 9.3$$

$$SNR_{a(t)} = 10 \log_{10} \left[\frac{\beta^2 P_{st}^2 R_L G^2}{2qB(BO + I_d) R_L G^2 + 2FkTB} \right] \quad 9.4$$

$$P_r = 10 \log_{10} \left[\frac{P_{st}(128)(4000)^2}{50 \times 10^{-3}} \right] dbm \quad 9.5$$

3.2.2.8 Variable Definitions

P_r = Power received from target (watts)

P_t = Power transmitted to target (watts)

R = Range to target (meters)

Ω_T = solid angle of transmitted beam in steradians where

$1^\circ = 0.3495 \text{ SR}$

ρ, δ = reflectance of target in fractional %

A_r = Area of target in (m^2)

A_o = clear aperture of receiving optics (m^2)

Ω_T = Solid angle of returned beam from target it may contain $\cos^2 \theta$, or Lambertian components being both specular and diffuse (SR)

T = Transmissivity of optical path in fractional part. This term is easily expanded to include atmospheric considerations and detailed optical losses as well as non-linear terms.

P_t = Power transmitted (watts)

E_t = Energy transmitted (Joules)

t = Length of pulse envelope (s)

nt = # of photons transmitted

h = Planck's constant (6.6×10^{-34} J)

c = speed of light $\sim 3 \times 10^8$ m/s

λ = wavelength (meters)

i(t), v(t) = photocurrent or voltage at detector

η = quantum efficiency of detector

q = electronic charge 1.6×10^{-19} C

f_a = mean frequency (Hz) defined by

$$f_a = \frac{f P_{lo} + P_s(t)}{P_{lo} P_s(t)}$$

P_{lo} = Power of local oscillator at detector this is derived from P_t/A_t (watts)

A_e = Attenuation factor, may be better modeled using known reflectivities, optical losses, BW and light leakage.

$P_r(t)$ = power of return signal over the duration of the return pulse but only the portion of signal which matches the polarization of the local oscillator P_{lo} , therefore $P_r(t) = P_r / P_{lo}$.

P_{ol} = Polarization attenuation factor as extinction coefficient

f_o = angular frequency of transmit signal ($f(2\pi)$) in (Hz).

f_b = Doppler shifted frequency back (Hz).

$t_{sub R}$ = Total pulse length of return signal (s).

ϕ_s = Phase shift of return signal as a multiple of π .

f = frequency of transmitted signal as $\frac{c}{\lambda}$ in (Hz)

V = velocity (m/s).

$\#(fringes)$ = Fringes seen by the detector (Hz).

db, dbm = S/N ratio in db's.

$W(t)$ = Power distribution of return signal over the detector area in (watts).

$\Delta(fringes)$ = # fringes shifted for a given velocity measurement in (Hz).

Δt = time delay or signal pulse broadening due to target velocity (m/s).

S/N = Signal to noise ratio (absolute).

axial mode = axial mode spacing for a given cavity length at a known wavelength λ , this is used for defining waveforms (Hz).

L = cavity length (m).

W_1 = axial mode spacing for 1 laser (Hz).

W_2 = axial mode spacing for a second laser, (Hz).

N_a = N # of modes superimposed on a signal

sinc = the function $\left(\frac{\sin \pi x}{x}\right)$.

t_1/a_1 = period of mode (Hz).

$\phi X, \theta x$ = phase shift of mode in cavity or external to the cavity.

Time = time of transit (flight of transmit signal until received) (s).

ΔV_E = Energy detection velocity resolution (m/s).

ΔV_H = Heterodyne detection velocity resolution, (m/s).

Δf_H = Heterodyne frequency resolution (Hz).

TP, T_p = Total sinusoidal envelope pulse length (s).

τ, t_x = fine structure of discrete pulse width inside of envelope TP (s).

B, BW = bandwidth for heterodyne case (Hz).

ΔR_E = Range resolution for the energy detection system (m).

ΔR_H = Range resolution for heterodyne system (m).

PT, P_T = A standard simple double pulse waveform description.

P_o = the average power level of waveform PT .

ΔR = Range resolution for Doppler imaging (m).

f_{di} = radar Doppler return from the i th target element (Hz).

v_i = velocity of the i th element inline with radar line of sight (m/s).

\vec{v}_i = velocity vector of each element $[x,y,z]$ in space (m/s)/(x,y).

$\vec{\Omega}$ = target rotation vector velocity (m/s).

\vec{r}_i = distance vector from rotation axis to the scattering element (m).

\vec{R} = Lidar range vector (m).

Δf_{di} = cross range increment of Doppler return.

ΔX_i = cross range increment (Hz).

T_m = measurement or sample time (s).

ΔX = cross range resolution (m).

$\Delta\theta$ = is the angle through which the target axis rotates during Doppler frequency measurement in time T_m .

f_1 = Doppler shifted frequency increment (Hz).

B_o = optical filter BW \AA° .

G = photodetector gain.

H_{rs} = solar spectral irradiance over spectral sensitivity window derived from blackbody calculation $\text{W m}^{-2} \text{\AA}^{-1}$.

H_s = Solar irradiance in window of interest at detector.

I_d = Detector dark current.

$K = 1.38 \times 10^{-23} \text{ J/K}^\circ$ (Boltzmann's constant).

R_L = Detector load resistance

$T = 300^\circ\text{K}$

T_t = Transmittance of transmitter optics

T_r = Transmittance of receiver optics.

X = optical filter leakage.

α_r = receive beamwidth.

Γ = Target reflectance

B = Detector responsivity (A/W).

σ = Atmospheric attenuation (km^{-1}).

σ_s = Backscatter coefficient (km^{-1}).

F = receiver noise factor.

P_t = Transmit power (watts)/ attenuation due to Tone 2
(if used).

$P_t(t-2R/c)$ = transmitted power

$B(R)$ = Backscatter coefficient.

η_h = heterodyne mixing efficiency

η_o = system optical efficiency.

$T(R)$ = Two way transmission losses.

$O(R)$ = range dependent overlap.

A_{det}/R^2 = Distance factor.

For the LP-140 we find $\nu_o = 28.28$ THz, $\Delta_{max} = 149$ MHz,
 $\Delta\nu_d = 53.7$ MHz (linewidth), $\Delta\nu_c = 274$ MHz. At 50
torr the LP-140 has a line width of 299×10^6 Hz. and a
coherence length of 1052 Km. [11, 10 , 8]

4 INITIAL CHARACTERIZATION

4.1 Pulse energy

The initial pulse energy for the system was measured as approximately 1 joule per pulse at 1 Hz. The pulse energy stability was poor and varied not only from shot to shot but decreased to below a joule after only a few minutes of operation. As the repetition rate was increased the average pulse power decreased. During the test it was discovered that a

minimum of 40 minutes was required for the operational stability of the system to be reached. The output energy is consistently lower after a short run period than immediately after startup. In the cool system the energy varied by 10% from shot to shot, while after warm up the energy varied by < 5%. [1, 2, 3]

4.2 Temporal profile

The temporal profile was measured using a PSI fast response pyroelectric detector, Model # UF-1 with a responsivity of 10^{-7} V/W. In order to prevent detector damage the beam was attenuated before being focussed onto the detector.

Significant mode beating was observed during the first few microseconds of operation. [1]

4.3 Spatial profile

The spatial profile was measured using a Spiricon beam profiler and the laser transverse mode pattern was found to be highly unstable with a large degree of beam pointing instability. Local hot spots were observed in the beam distribution which would vary from shot to shot. This indicated transverse mode hopping. One particularly interesting phenomena was the observation of the spot of Arago (Poisson spot), as a linear zone extended across the obscured output of the laser. The band was always found 1/3 of the way from the nearest face of the obscuration and seemed never to bisect the

exact center.

This spike occurs because of a collection of scattered Fresnel diffraction terms near the hard obscuring edge of the mirror. The diffraction function describing this is shown as [16]

$$\bar{u}(r, z) \approx -\frac{\bar{q}0}{\bar{q}(z)} e^{-j\pi N - a^2/\omega_0^2} \times e^{-j\pi N(r/a)^2} J_0(2\pi N r/a) \quad 9.5$$

This expression produces a central spike of amplitude $\approx \sigma_a$, and surrounding Bessel function rings, which are present everywhere along the axis near the center of the shadow. This spot contains very little energy and was quite transient.

4.4 Frequency chirp

The intrapulse frequency chirp was measured by feeding the laser output into the existing MSFC lidar. The data was collected on the 10P22 and 10P20 line. Due to transverse mode instabilities it was difficult to obtain measurement but the chirp varied in excess of 8 MHz and varied considerably from shot to shot. It was observed that the LP-140 had far more energy output on the 10P22 line than the 10P20 line. On 10P20 only 0.65 joules per pulse was obtained, while on the 10P22 line 1.0 Joules was obtained. The laser was also prominent on the 10P30 line as well. [1, 2]

5 SYSTEM MODIFICATIONS

5.1 Gas flow

The degradation in output energy with both increased pulse rate and number of pulses tends to indicate insufficient gas flow through the laser. This results in pressure, thermal, and possible slight dissociation mechanisms in the gas leading to the reduced output. During operation, the needle on the pressure gauge momentarily rises by as much as 10 torr. This is indicative of the pressure wave which is generated inside the laser discharge.

Inspection of the laser head shows that the gas in and out flow are limited by the orifice size of the flow tubing and the existence of a micron gas filter in line on the inlet side of the system. The standard LP-140 laser flows gas through the discharge at a trickle rate of 0.5 L/min. The gas flow was modified by increasing the diameter of the input and output lines and reversing the direction of flow in the laser head. This enabled the flow rate to be increased to 2.5 L/min. [1, 3]

During this investigation the laser was held to lase on the 10P(20) line. A bottled premixed gas was used to avoid problems associated with constructing a gas specific calibrated gas flow manifold and residual gas analyzer. This enabled all measurements to be done with a homogeneous gas mix. Measure-

ments were made at the following gas pressures: 20, 30, 35, 40, 45, 50, and +50 torr within the laser head with a gas flow rate of 2.5 liters per minute. An additional experiment was run with the laser in a sealed (non-gas flowing) configuration at 45 torr. Two sets of data were collected at each pressure, first one without an injection signal and then one with an injection signal. The injection signal was obtained from an optogalvanically stabilized waveguide cw laser. The laser was allowed to stabilize at each pressure for 10 minutes before the data was collected. The data was sampled using a Hewlett Packard time and frequency analyzer and downloaded to an IBM compatible computer and placed in ASCII files. Each file represented a single chirp trace and consisted of 100 data points. These files were analyzed using a MathCad document and BASIC program to do conventional statistical analysis.

It was determined that the laser could be operated satisfactorily with a stable discharge and good output energy stability in a sealed fashion for up to 1×10^6 shots. There was also definite slight increase in average output energy noticed after the laser had reached thermal equilibrium. It is felt that the gas volume over time reaches thermal equilibrium and various eddy currents due to the localized heating of the gas reach a state of stable dissipation over time. As the gas volume warms up the surrounding resonator also reaches thermal

equilibrium and convection with ambient air is balanced. Once this has occurred a catalyst action from the gold on the plated electrodes refurbishes the laser gas every few hundred shots or whenever a random arc occurs, this mechanism allows the gas volume to remain replenished and recycled shot to shot. The experiment was performed at 1 Hz and it is believed that at higher repetition rates the existing catalyst action may not be able to keep up with the gas disassociation but this was not verified. [2]

5.2 Cavity length

The cavity was lengthened by increasing the spacing between the grating and the fourth optical element (plano-convex lens). This enabled a more stable operation point to be obtained. Additionally the increased path length reduces spurious off-axis modes. The cavity was extended mechanically in 2.54 cm increments. Previously, the grating would have to be adjusted every two to three shots to maintain a uniform laser output. For an extension 53 +/- 5 cm the distribution became very uniform and stable. The optimum beam stability and mode patterns however, did not coincide. The energy increased to about 1.25 joules for an extension of 51 cm but there remained a slight degree of mode instability characterized by the occasional appearance of a hot spot in the beam profile. [1]

At 54 cm extension the beam pointing instability was minimized. At 56 cm the energy decreased to 1.15 joules and the mode distribution was at its most stable point. The output quickly became chaotic as the cavity extension decreased below 45 cm. There was also an accompanying energy drop to 0.75 joules for further extensions below 25 cm. As the cavity spacing was further decreased the energy once again increased. For extensions above 58 cm the power dropped off linearly. Mode and pointing instability never set in for cavity extensions out to 90 cm. Although stable, the output energy distribution collapsed to fill only a single corner of the possible output mode pattern. This corner remained very stable in distribution but fluctuated in energy. For extensions beyond 75 cm it became impossible to fill the output pattern despite adjustments to the grating for alignment.

During this effort it was determined that rotating the grating by 90 degrees decreased the output energy by 50-75%. The output coupler was also rotated by 45 degrees to observe the diffraction perturbation. The rotated output coupler resulted in a unique near field pattern that seemed to superimpose three copies of the central output pattern slightly in the horizontal axis. It did not simply rotate the axis of the output pattern as might be expected initially. [1]

The plano-convex lens was also replaced with a flat uncoated NaCl window. The laser operated with a stable output energy and spatial distribution. The energy however decreased by 50%. The greater portion of this is accounted for by the lack of A.R. coatings, and the poor optical quality of the substrate. We are quite confident that the increased diffraction losses caused by operating the laser at a higher magnification using the flat window will be sufficient to overcome the spatial mode instabilities found with the shorter cavity length.

6 FINAL CHARACTERISTICS

6.1 Pulse Energy

The pulse energy was immediately observed to have increased slightly and stabilized. At the most stable point for pulse to pulse repeatability, the energy is equal to the original performance of 1 J/pulse. When the cavity length is slightly modified from this point by an extension of 2-5 cm, the energy increases by 10-15%. The spatial profile however is not as stable.

The optimum point of operation is better described as a window about 11 cm wide peaked at a cavity length extended by 56 cm.

[1, 2, 3]

6.2 Temporal profile

The temporal profile at the peak cavity length improves substantially. The pulse length not only shortens, but the tendency for multi-pulsing is also reduced.

At the original cavity length, inconsistent pulse lengths up to 12 microseconds were not uncommon with triple spikes or mode hops in the pulse. By extending the cavity, these pulses have been reduced to 5-8 microseconds with a single well defined peak. It is important to note that at all cavity spacings, the gas mixture and pressure also played an important role in defining the temporal profile. [1, 2]

6.3 Spatial profile

The spatial profile at the optimal cavity spacing demonstrates the most significant improvement. Outside of the 11 cm window the profile is chaotic and exhibited a strong tendency toward beam pointing instability. Within the window the temporal profile fills out and the beam settles down to a high degree of stability. [1, 2]

It is difficult to quantify the exact nature of the spatial mode. The center obscuration creates a square output beam with a square hole in the center. The far field diffraction pattern is non-uniform and is not adequately described as a gaussian or top hat. Both the near and far-field patterns display a large amount of diffraction ripple.

6.4 Frequency chirp

The chirp data was taken for pressures ranging from 30-50 torr and was determined to be repeatable with a significant dependence on pressure. The higher pressure range from 40-50 torr provided the best results. The frequency chirp shows that the initial downsweep in frequency can be attributed to the decaying electron density in the discharge while the upswing is the conventional t^3 dependence. The average pulse length was about 13 microseconds initially, later this was effectively reduced to 5 microseconds especially when running the laser in a sealed condition. The total chirp over the duration of the pulse was shown to be about 2 MHz. The chirp however was seen to be as low as 250 KHz during a 2 - 5 microsecond temporal slice within the pulse. [1, 4, 5]

The chirp was expected and found to be highly dependent on gas pressure and mixture. The N_2 and He especially have a dramatic effect on varying both the pulse length and the chirp. The chirp is directly related to the pulse energy. With lower pulse energies producing the lowest chirp. If the gas flow and mixture is varied the expected chirp follows almost linearly with the final output energy of the laser pulse.

6.5 Discharge Characteristics

Each of the discharges is driven by a separate circuit, although they are all switched by a common EG&G HY1102 thyra-

tron. The discharge voltage pulse was measured using a Pearson Electronics model 110 Induction coil. Both were connected to a LeCroy 9450 digitizing oscilloscope which downloaded the signal pulses to an IBM PC computer for analysis. Typical current and voltage pulses for one of the electrodes was measured and this relates approximately to the other electrode pairs in the laser. It was found that there exists considerable ringing on the voltage pulse which was determined to be due to a mismatch in the impedance of the discharge circuit in the laser. [4, 5]

The electrical energy into the discharge can be obtained by integrating the voltage and current pulses. The energy deposited into a single electrode pair was found to be 3.5 J. However, only the energy deposited into the $\text{CO}_2(001)$ and the $\text{N}_2(v=1-8)$ vibrational energy levels is useful for lasing action. It has been shown in the literature that the optimum excitation of these vibrational levels occurs for a reduced electric field or E/N of $1-3 \times 10^{-16} \text{ Vcm}^2$. The value of E/N was derived from the discharge voltage pulse form for each digitization interval together with the energy input in that interval. This enabled the input energy into the discharge to be evaluated as a function of E/N for E/N intervals of $1 \times 10^{-16} \text{ Vcm}^2$. We found that most of the energy is deposited in the $4-6 \times 10^{-16} \text{ Vcm}^2$. This results in a lower laser effi-

ciency of 4% as compared to a theoretical yield of 12% for the laser in its original multimode configuration. This implies that the discharge circuit requires optimizing for the new laser gas mixture. The evolution of the discharge impedance with time was found by dividing the voltage by the current. We found that the discharge impedance during the bulk of the energy input varies between 5Ω and 20Ω , this enables the circuit to be optimized. [4, 5]

6.6 Discharge Analysis

The voltage was measured by Spiers and Thistlethwaite (1991), using a Tektronix P6015 high voltage probe connected to a LeCroy 9450 dual 350 MHz digitising oscilloscope. The discharge current pulse was measured simultaneously using a Pearson Electronics Model 110 induction coil. After being digitized by the oscilloscope, the pulses were either ported to a plotter or IBM compatible computer for analysis. [4]

The collected data demonstrated that there exists a prominent voltage rise to 4.5 kV. This represents the preionisation pulse which is followed by the primary discharge pulse which abruptly rises to 5.5 kV. The voltage pulse then continues to ring across the discharge with an average value of 4.5 kV. The subsequent ringing is not too severe but is undesirable and emphasizes the need for impedance matching of the discharge circuit. The current was seen to peak at a value 650 A as the

voltage begins to drop off.

Further analysis was accomplished by Spiers who developed analytical code to determine the discharge impedance, electron concentration, electron drift velocity, and energy deposition as a function of time. [4, 5]

7 INJECTION SEEDING

During this development effort it was discovered that the LP-140 would operate with a high degree of spatial mode stability of an aperture was inserted between optical elements #4 and #5 (the plano-convex lens and grating). The aperture is square with rounded corners and reduced the actual aperture optically by 5%. The reasoning behind using this aperture was the unusual characteristics of the laser energy with cavity extension. The unusual dip that occurs corresponds with the distance required for the intra cavity beam to expand by diffraction enough to start clipping the edge of the grating, beyond that point the beam over fills the grating and is vignetted. Since the vignetting suppresses off axis parasitic mode oscillations it was reasoned that maintaining a short cavity but simulating the vignetting may produce the same stable output. This concept was verified experimentally. The amount of over fill was calculated to be 5% at the optimal cavity extension, and the beam aperture was reduced by the corresponding amount by cutting a square hole in

a piece of plexi-glass of appropriate dimensions. The optical output then matched those of the extended cavity both in output energy, beam stability and pointing stability. [1, 2, 8] All injection experiments were carried out on the 10P(20) line.. The gas mixture used was from a premixed cylinder composed of CO₂:N₂:CO:He in the proportions of 15.29:17.33:1.99:65.39 with a 5% tolerance.

The collected data exhibited a fair amount of noise superimposed on the chirp data. This is probably due to RF noise transients superimposed on the signal and detector. Evidence that this was occurring arose from the fact that the noise was high enough to trigger the scope sweep even without a trigger signal. Earlier experiments were performed with the laser in a metal housing while this one was done with the laser exposed for engineering purposes. The data was sampled with only 100 discrete points and this made FFT filtering to remove the noise difficult. Alternatively a manual curve fit was done to the data and a statistical analysis performed on the results.

Because there are pulse to pulse variations in the data a method of comparing all the data sets was required. The laser induced medium perturbation (limp) is considered as:[10, 8, 6, 4, 1]

$$f(t) = \frac{\alpha E t^3}{\tau} \quad 9.6$$

where α is defined in $\text{kHz}\mu\text{s}^{-2}\text{J}^{-1}$ is a constant, E is the pulse energy, τ is the pulse length and t is the time. Therefore for each pulse a value of α can be obtained, thereby enabling a valid comparison to be made. The alpha coefficients obtained for the various configurations are as follows:

CONFIGURATION	α	$(\tau)_{\mu s}$	$f(\tau)(MHz)$
45 torr flowing uninjected	69	6.5	0.97
45 torr flowing injected	27	7.5	0.78
45 torr sealed uninjected	90	5.0	1.40
45 torr sealed injected	20	6.0	0.68
50 torr sealed uninjected	240	4.1	0.86
50 torr sealed flowing injected	95	5.45	0.75

[2]

Previous investigations had indicated that the laser operated with a stable discharge and good power stability in a sealed fashion for up to 1×10^6 shots. It was also noted that as the pressure decreased below 40 torr the pulse length increased beyond 15 microseconds and the chirp decreased accordingly with the energy which was measured as 750 mJ. At 45 torr the pulse length varied between 10 and 15 microseconds, which is in good agreement with earlier experiments. At this pressure the energy was 480 mJ. A single 2 to 5 microsecond, segment taken from the pulse averages 450 kHz of frequency perturbation. Data taken at 45 torr sealed indicated that the energy increased to 500 mJ with a pulse length shortening to 5 microseconds over about 10

shots. When the pressure was increased to 50 torr the pulse shortened dramatically to a well defined 5 microseconds, with a total pulse energy of only 200 mJ. It was also noted that at 50 torr the discharge exhibited substantial arcing. This is probably due to gross mechanical misalignment of the discharge electrodes by the manufacturer, consequently much of the pump energy is lost in the arc resulting in reduced energy and substantial RF noise. [2]

7.1 Temporal profile

The temporal profile was measured using a fast response SAT detector to correlate with the chirp signals obtained. The agreement was good and indicated that the pulse length and shape at each pressure agreed in both measurement schemes. The two most significant data sets were taken at 50 torr flowing with injection seeding and at 45 torr sealed without injection seeding. These two data sets are similar in temporal profile, energy, and stability.

8 CONCLUSION

The initial engineering goal of improving the LP-140 operationally has been realized and the development summary is shown below:

[1, 2]

MODIFICATION SUMMARY		
BASELINE CHARACTERIS- TICS	MODIFICATIONS	FINAL CHARACTERIS- TICS
Severe Transverse mode hopping	Inserted 5% reduc- ing aperture	Transverse mode sta- bility
Unstable output energy (1 J/pulse)	Sealed laser	Stable operation with slight energy increase (1.2 J/pulse)
Long optical pulse (10 - 30 μ s)	Changed gas mix and increased pressure	5 μ s pulse
Poor interpulse and intrapulse frequency stability (> 8 MHz)	Injected cw laser	Good chirp (< 3 MHz)

Injection seeding does have a chirp reducing effect for both interpulse and intrapulse frequency perturbation. The pulse lengths and chirp profiles demonstrated in earlier experiments were verified. The LP-140 seemed to produce the best results in a sealed configuration. The final laser characteristics although substantially improved fall short of the required 2 microsecond pulses with < 200 kHz chirp with 1 J/ pulse energies up to repe-

tition rates of 7 Hz. Since these are the absolute driving criteria it is safe to conclude that the LP-140 is inherently incapable of achieving those performance specifications and as such should not be considered further by MSFC as a viable coherent LIDAR candidate. The laser does have outstanding qualities for a direct energy detection system and may find useful benefit as an inexpensive light source for such a system especially where low ambient noise levels are present and the theoretical S/N improvement from such a system can be realized.

9 APPENDIX

[6, 9, 13, 14, 15]

9.1 Source code

9.1.1 QuickBasic ver 4.5

The following source code was prepared as a tool in doing signal processing calculations. The following listings test several different FFT algorithms from either a test function or the function may be replaced by the digitized data files.

[13]

9.1.1.1 FFTs

[13]

```
DECLARE SUB PRNTFFT (DATQ!(), NN2!)
```

```
DECLARE SUB FOUR1 (DATQ!(), NN!, ISIGN!)
```

```

'PROGRAM D12R1

'DRIVER for routine FOUR1

CLS : SCREEN 9

NN = 32

NN2 = 2 * NN

DIM DATQ(NN2), DCMP(NN2)

PRINT "h(t) = real valued even function"

PRINT "H(n) = H(N-n) and real?"

PRINT

FOR I = 1 TO 2 * NN - 1 STEP 2
    DATQ(I) = 1! / (((I - NN - 1!) / NN) ^ 2 + 1!)
    DATQ(I + 1) = 0!
NEXT I

ISIGN = 1

CALL FOUR1(DATQ(), NN, ISIGN)

CALL PRNTFT(DATQ(), NN2)

PRINT "h(t) = imaginary - valued even-function"

PRINT "H(n) = H(N-n) and imaginary?"

PRINT

FOR I = 1 TO 2 * NN - 1 STEP 2
    DATQ(I + 1) = 1! / (((I - NN - 1!) / NN) ^ 2 + 1!)
    DATQ(I) = 0!
NEXT I

ISIGN = 1

```

```

CALL FOUR1(DATQ(), NN, ISIGN)
CALL PRNTFT(DATQ(), NN2)
PRINT "h(t) = real valued odd function"
PRINT "H(n) = -H(N-n) and imaginary?"
PRINT
FOR I = 1 TO 2 * NN - 1 STEP 2
    DATQ(I) = (I - NN - 1!) / NN / (((I - NN - 1!) / NN) ^
2 + 1!)
    DATQ(I + 1) = 0!
NEXT I
DATQ(1) = 0!
ISIGN = 1
CALL FOUR1(DATQ(), NN, ISIGN)
CALL PRNTFT(DATQ(), NN2)
PRINT "h(t) = imaginary valued odd function"
PRINT "H(n) = -H(N-n) and real?"
PRINT
FOR I = 1 TO 2 * NN - 1 STEP 2
    DATQ(I + 1) = (I - NN - 1!) / NN / (((I - NN - 1!) /
NN) ^ 2 + 1!)
    DATQ(I) = 0!
NEXT I
DATQ(2) = 0!
ISIGN = 1

```

```

CALL FOUR1(DATQ(), NN, ISIGN)
CALL PRNTFT(DATQ(), NN2)
'TRANSFORM INVERSE-TRANSFORM TEST
FOR I = 1 TO 2 * NN - 1 STEP 2
    DATQ(I) = 1! / ((.5 * (I - NN - 1) / NN) ^ 2 + 1!)
    DCMP(I) = DATQ(I)
    DATQ(I + 1) = (.25 * (I - NN - 1) / NN) * EXP(-(.5 * (I
- NN - 1!) / NN) ^ 2)
    DCMP(I + 1) = DATQ(I + 1)
NEXT I
ISIGN = 1
CALL FOUR1(DATQ(), NN, ISIGN)
ISIGN = -1
CALL FOUR1(DATQ(), NN, ISIGN)
PRINT "          ORIGINAL DATA:          DOUBLE FOURIER
TRANSFORM:"
PRINT
PRINT "    K          REAL h(K)      IMAG h(K)      REAL h(K)
IMAG h(K)"
PRINT
FOR I = 1 TO NN STEP 2
    J = (I + 1) / 2
    PRINT USING "####"; J;
    PRINT USING "#####.#####"; DCMP(I); DCMP(I + 1);

```



```

DATQ(I) / NN; DATQ(I + 1) / NN
NEXT I
INPUT dum$
END
SUB FOUR1 (DATQ(), NN, ISIGN)
N = 2 * NN
J = 1
FOR I = 1 TO N STEP 2
    IF J > I THEN
        TEMPR = DATQ(J)
        TEMPI = DATQ(J + 1)
        DATQ(J) = DATQ(I)
        DATQ(J + 1) = DATQ(I + 1)
        DATQ(I) = TEMPR
        DATQ(I + 1) = TEMPI
    END IF
    M = INT(N / 2)
    WHILE M >= 2 AND J > M
        J = J - M
        M = INT(M / 2)
    WEND
    J = J + M
NEXT I
MMAX = 2

```

```

WHILE N > MMAX
    ISTEP = 2 * MMAX
    THETA# = 6.28318530717959# / (ISIGN * MMAX)
    WPR# = -2# * SIN(.5# * THETA#) ^ 2
    WPI# = SIN(THETA#)
    WR# = 1#
    WI# = 0#
    FOR M = 1 TO MMAX STEP 2
        FOR I = M TO N STEP ISTEP
            J = I + MMAX
            TEMPR = CSNG(WR#) * DATQ(J) - CSNG(WI#) * DATQ(J
+ 1)
            TEMPI = CSNG(WR#) * DATQ(J + 1) + CSNG(WI#) *
DATQ(J)
            DATQ(J) = DATQ(I) - TEMPR
            DATQ(J + 1) = DATQ(I + 1) - TEMPI
            DATQ(I) = DATQ(I) + TEMPR
            DATQ(I + 1) = DATQ(I + 1) + TEMPI
        NEXT I
        WTEMP# = WR#
        WR# = WR# * WPR# - WI# * WPI# + WR#
        WI# = WI# * WPR# + WTEMP# * WPI# + WI#
    NEXT M

```

```

        MMAX = ISTEP
    WEND

END SUB

SUB PRNTFT (DATQ(), NN2)
    PRINT "      n      REAL H(n)      IMAG H(n)      REAL H(N-n)
    IMAG H(N-n)"

    PRINT

    PRINT USING "####"; 0;

    PRINT USING "#####.#####"; DATQ(1); DATQ(2); DATQ(1);
    DATQ(2)

    FOR N = 3 TO NN2 / 2 + 1 STEP 2

        M = (N - 1) / 2

        MM = NN2 + 2 - N

        PRINT USING "####"; M;

        PRINT USING "#####.#####"; DATQ(N); DATQ(N + 1);
    DATQ(MM); DATQ(MM + 1)

    NEXT N

LINE INPUT dum$

CLS

END SUB

```

9.1.1.2 FFT2

[13]

```
DECLARE SUB PRNTFT (DATQ!(), N2!)
```

```
DECLARE SUB TWOFFT (DATA1!(), DATA2!(), FFT1!(), FFT2!(),  
N!)
```

```
DECLARE SUB FOUR1 (DATQ!(), NN!, ISIGN!)
```

```
'PROGRAM D12R2
```

```
'DRIVER FOR ROUTINE TWOFFT
```

```
CLS : SCREEN 9
```

```
N = 32
```

```
N2 = 2 * N
```

```
PER = 8!
```

```
PI = 3.14159
```

```
DIM DATA1(N), DATA2(N), FFT1(N2), FFT2(N2)
```

```
FOR I = 1 TO N
```

```
    X = 2! * PI * I / PER
```

```
    DATA1(I) = INT(COS(X) + .5)
```

```
    DATA2(I) = INT(SIN(X) + .5)
```

```
NEXT I
```

```
CALL TWOFFT(DATA1(), DATA2(), FFT1(), FFT2(), N)
```

```
PRINT "      FOURIER TRANSFORM OF FIRST FUNCTION:"
```

```
PRINT
```

```
CALL PRNTFT(FFT1(), N2)
```

```
PRINT "      FOURIER TRANSFORM OF SECOND FUNCTION:"
```

```

PRINT
CALL PRNTFT(FFT2(), N2)
'INVERT TRANSFORM
ISIGN = -1
CALL FOUR1(FFT1(), N, ISIGN)
PRINT "    INVERTED TRANSFORM = FIRST FUNCTION:"
PRINT
CALL PRNTFT(FFT1(), N2)
CALL FOUR1(FFT2(), N, ISIGN)
PRINT "    INVERTED TRANSFORM = SECOND FUNCTION:"
PRINT
CALL PRNTFT(FFT2(), N2)
END
SUB FOUR1 (DATQ(), NN, ISIGN)
N = 2 * NN
J = 1
FOR I = 1 TO N STEP 2
    IF J > I THEN
        TEMPR = DATQ(J)
        TEMPI = DATQ(J + 1)
        DATQ(J) = DATQ(I)
        DATQ(J + 1) = DATQ(I + 1)
        DATQ(I) = TEMPR
        DATQ(I + 1) = TEMPI
    
```

```

END IF

M = INT(N / 2)

WHILE M >= 2 AND J > M

J = J - M

M = INT(M / 2)

WEND

J = J + M

NEXT I

MMAX = 2

WHILE N > MMAX

    ISTEP = 2 * MMAX

    THETA# = 6.28318530717959# / (ISIGN * MMAX)

    WPR# = -2# * SIN(.5# * THETA#) ^ 2

    WPI# = SIN(THETA#)

    WR# = 1#

    WI# = 0#

    FOR M = 1 TO MMAX STEP 2

        FOR I = M TO N STEP ISTEP

            J = I + MMAX

            TEMPR = CSNG(WR#) * DATQ(J) - CSNG(WI#) * DATQ(J
+ 1)

            TEMPI = CSNG(WR#) * DATQ(J + 1) + CSNG(WI#) *
DATQ(J)

            DATQ(J) = DATQ(I) - TEMPR

```

```

        DATQ(J + 1) = DATQ(I + 1) - TEMPI
        DATQ(I) = DATQ(I) + TEMPR
        DATQ(I + 1) = DATQ(I + 1) + TEMPI
    NEXT I

    WTEMP# = WR#

    WR# = WR# * WPR# - WI# * WPI# + WR#
    WI# = WI# * WPR# + WTEMP# * WPI# + WI#

    NEXT M

    MMAX = ISTEP

WEND

END SUB

SUB PRNTFT (DATQ(), N2)
PRINT "      n      REAL(n)      IMAG(n)      REAL(N-n)
      IMAG(N-n)"
PRINT
PRINT USING "#####"; 0;
PRINT USING "#####.#####"; DATQ(1); DATQ(2); DATQ(1);
      DATQ(2)
FOR I = 3 TO N2 / 2 + 1 STEP 2
    M = (I - 1) / 2
    NN2 = N2 + 2 - I
    PRINT USING "#####"; M;
    PRINT USING "#####.#####"; DATQ(I); DATQ(I + 1);
      DATQ(NN2); DATQ(NN2 + 1)

```

```

NEXT I

LINE INPUT DUM$

CLS

END SUB

'DECLARE SUB FOUR1 (DATQ!(), NN!, ISIGN!)

SUB TWOFFT (DATA1(), DATA2(), FFT1(), FFT2(), N)

C1R = .5

C1I = 0!

C2R = 0!

C2I = -.5

FOR J = 1 TO N

    FFT1(2 * J - 1) = DATA1(J)

    FFT1(2 * J) = DATA2(J)

NEXT J

CALL FOUR1(FFT1(), N, 1)

FFT2(1) = FFT1(2)

FFT2(2) = 0!

FFT1(2) = 0!

N2 = 2 * (N + 2)

FOR J = 2 TO N / 2 + 1

    J2 = 2 * J

    CONJR = FFT1(N2 - J2 - 1)

    CONJI = -FFT1(N2 - J2)

    H1R = C1R * (FFT1(J2 - 1) + CONJR) - C1I * (FFT1(J2) +

```



```

CONJI)
    H1I = C1I * (FFT1(J2 - 1) + CONJR) + C1R * (FFT1(J2) +
CONJI)
    H2R = C2R * (FFT1(J2 - 1) - CONJR) - C2I * (FFT1(J2) -
CONJI)
    H2I = C2I * (FFT1(J2 - 1) - CONJR) + C2R * (FFT1(J2) -
CONJI)

    FFT1(J2 - 1) = H1R
    FFT1(J2) = H1I
    FFT1(N2 - J2 - 1) = H1R
    FFT1(N2 - J2) = -H1I
    FFT2(J2 - 1) = H2R
    FFT2(J2) = H2I
    FFT2(N2 - J2 - 1) = H2R
    FFT2(N2 - J2) = -H2I

    NEXT J

END SUB

9.1.1.3 FFT3

[ 13 ]

DECLARE SUB REALFT (DATQ!(), N!, ISIGN!)
DECLARE SUB FOUR1 (DATQ!(), NN!, ISIGN!)

'PROGRAM D12R3

'DRIVER FOR ROUTINE REALFT, CALCULATES FFT OF A SINGLE
REAL VALUED ARRAY

```

```

CLS : SCREEN 9

EPS = .001

NP = 128

WIDTQ = 50!

PI = 3.14159

DIM DATQ(NP), SIZE(NP)

N = NP / 2

DO

    PRINT "PERIOD OF SINUSOID IN CHANNELS (2 -"; NP; ";
OR 0 TO STOP)"

    INPUT PER

    IF PER <= 0! THEN EXIT DO

    FOR I = 1 TO NP

        DATQ(I) = COS(2! * PI * (I - 1) / PER)

    NEXT I

    CALL REALFT(DATQ(), N, 1)

    SIZE(1) = DATQ(1)

    BIG = SIZE(1)

    FOR I = 2 TO N

        SIZE(I) = SQR(DATQ(2 * I - 1) ^ 2 + DATQ(2 * I)
^ 2)

        IF I = 1 THEN SIZE(I) = DATQ(I)

        IF SIZE(I) > BIG THEN BIG = SIZE(I)

    NEXT I

```

```

SCAL = WIDTQ / BIG
FOR I = 1 TO N
    NLIM = INT(SCAL * SIZE(I) + EPS)
    PRINT USING "####"; I;
    PRINT " ";
    FOR J = 1 TO NLIM + 1
        PRINT "*";
    NEXT J
    PRINT
NEXT I
PRINT "=>"
LINE INPUT DUM$
CLS
CALL REALFT(DATQ(), N, -1)
BIG = -1E+10
SMALL = 1E+10
FOR I = 1 TO NP
    IF DATQ(I) < SMALL THEN SMALL = DATQ(I)
    IF DATQ(I) > BIG THEN BIG = DATQ(I)
NEXT I
SCAL = WIDTQ / (BIG - SMALL)
FOR I = 1 TO NP
    NLIM = INT(SCAL * (DATQ(I) - SMALL) + EPS)
    PRINT USING "####"; I;

```

```

        PRINT " ";
        FOR J = 1 TO NLIM + 1
            PRINT "*";
        NEXT J
        PRINT
    NEXT I

LOOP
END
'
SUB FOUR1 (DATQ(), NN, ISIGN)
N = 2 * NN
J = 1
FOR I = 1 TO N STEP 2
    IF J > I THEN
        TEMPR = DATQ(J)
        TEMPI = DATQ(J + 1)
        DATQ(J) = DATQ(I)
        DATQ(J + 1) = DATQ(I + 1)
        DATQ(I) = TEMPR
        DATQ(I + 1) = TEMPI
    END IF
M = INT(N / 2)
WHILE M >= 2 AND J > M
    J = J - M

```

```

M = INT(M / 2)

WEND

J = J + M

NEXT I

MMAX = 2

WHILE N > MMAX

    ISTEP = 2 * MMAX

    THETA# = 6.28318530717959# / (ISIGN * MMAX)

    WPR# = -2# * SIN(.5# * THETA#) ^ 2

    WPI# = SIN(THETA#)

    WR# = 1#

    WI# = 0#

    FOR M = 1 TO MMAX STEP 2

        FOR I = M TO N STEP ISTEP

            J = I + MMAX

            TEMPR = CSNG(WR#) * DATQ(J) - CSNG(WI#) * DATQ(J
+ 1)

            TEMPI = CSNG(WR#) * DATQ(J + 1) + CSNG(WI#) *
DATQ(J)

            DATQ(J) = DATQ(I) - TEMPR

            DATQ(J + 1) = DATQ(I + 1) - TEMPI

            DATQ(I) = DATQ(I) + TEMPR

            DATQ(I + 1) = DATQ(I + 1) + TEMPI

        NEXT I

```

```

        WTEMP# = WR#

        WR# = WR# * WPR# - WI# * WPI# + WR#

        WI# = WI# * WPR# + WTEMP# * WPI# + WI#

    NEXT M

    MMAX = ISTEP

WEND

/

END SUB

SUB REALFT (DATQ(), N, ISIGN)

    THETA# = 3.141592653589793# / CDBL(N)

    C1 = .5

    IF ISIGN = 1 THEN

        C2 = -.5

        CALL FOUR1(DATQ(), N, 1)

    ELSE

        C2 = .5

        THETA# = -THETA#

    END IF

    WPR# = -2# * SIN(.5# * THETA#) ^ 2

    WPI# = SIN(THETA#)

    WR# = 1# + WPR#

    WI# = WPI#

    N2P3 = 2 * N + 3

    FOR I = 2 TO INT(N / 2)

```

```

      I1 = 2 * I - 1
      I2 = I1 + 1
      I3 = N2P3 - I2
      I4 = I3 + 1
      WRS# = CSNG(WR#)
      WIS# = CSNG(WI#)
      H1R = C1 * (DATQ(I1) + DATQ(I3))
      H1I = C1 * (DATQ(I2) - DATQ(I4))
      H2R = -C2 * (DATQ(I2) + DATQ(I4))
      H2I = C2 * DATQ(I1) - DATQ(I3)
      DATQ(I1) = H1R + WRS# * H2R - WIS# * H2I
      DATQ(I2) = H1I + WRS# * H2I + WIS# * H2R
      DATQ(I3) = H1R - WRS# * H2R + WIS# * H2I
      DATQ(I4) = -H1I + WRS# * H2I + WIS# * H2R
      WTEMP# = WR#
      WR# = WR# * WPR# - WI# * WPI# + WR#
      WI# = WI# * WPR# + WTEMP# * WPI# + WI#
NEXT I
IF ISIGN = 1 THEN
      H1R = DATQ(1)
      DATQ(1) = H1R + DATQ(2)
      DATQ(2) = H1R - DATQ(2)
ELSE
      H1R = DATQ(1)

```

```

    DATQ(1) = C1 * (H1R + DATQ(2))
    DATQ(2) = C1 * (H1R - DATQ(2))
    CALL FOUR1(DATQ(), N, -1)

END IF

END SUB

```

9.1.2 HP 48SX

The following listing may be used with the multiple equation solver found in the HP-equation library card for the HP-48SX hand held computer. The listing runs the lidar model presented under the extended theory sub heading. [13, 14, 6]

9.1.2.1 Lidar model

```

%%HP: T(1)A(R)F(.);

DIR

    MODESPACE 'w(m)=2
    *L/CONST(c)'

    AXMODE 'w(Hz)=
    CONST(c)/(2*L)'

    STARTsolver

    ½ IMG iLCD 7

FREEZE { EQ1 EQ2
EQ3 EQ4 EQ5 EQ6 EQ7
EQ8 EQ9 EQ10 EQ11
EQ12 EQ13 EQ14 EQ15
EQ16 EQ17 EQ18 EQ19

```



```
EQ20 EQ21 EQ22 EQ23
EQ24 EQ25 EQ26 EQ27
EQ28 EQ29 } 'EQ'
STO MINIT MSOLVR
```

```
¶
```

```
CLEANvars
```

```
½ ( Imt
```

```
çfringes fringes ò2
t1 ò1 a2 w2 N2 I2
a1 w1 N1 I1 L î t w
N db SN W IERR ¥t
EQ T R ¥r ¥T Ac Ar
Æ Pt Pr no nt Er Et
h û c t Mpar PPAR c
f it Pst òst fb Plo
fo v Pol At ö Ps òs
çt fa tR } PURGE
```

```
¶
```

```
EQ1 'Pr=Pt/(R^2*(
¥t*.0349065))*(Æ*Ar
)*(Ac/(R^2*(¥r*
.0349065)))*T^2'
EQ2 'Pt=Et/t'
EQ3 'Er=Pr*t'
```

$$\text{EQ4 } 'nt=Et/(CONST(h)*CONST(c)/\hat{u})'$$

$$\text{EQ5 } 'no=Er/(CONST(h)*CONST(c)/\hat{u})'$$

$$\text{EQ6 } 'it=\ddot{o}*CONST(q)/(CONST(h)*fa)*(Plo+Ps+2*(Plo*Ps)^{(1/2)}*COS((fo-fb*(2*\varsigma))*tR+\ddot{o}s))'$$

$$\text{EQ7 } 'fo=CONST(c)/\hat{u}*2*\varsigma'$$

$$\text{EQ8 } 'Plo=Pt/At'$$

$$\text{EQ9 } 'Ps=Pr/Pol'$$

$$\text{EQ10 } 'fb=2/\hat{u}*V+f'$$

$$\text{EQ11 } 'fringes=2*V/\hat{u}'$$

$$\text{EQ12 } 'db=10*LOG(\ddot{o}*Pr/(CONST(h)*f))'$$

$$\text{EQ13 } 'fa=(f*Plo+fb*Ps)/(Plo+Ps)'$$

$$\text{EQ14 } 'W=2*(\ddot{o}^2*CONST(q)^2/(CONST(h)*f^2))*Plo*\ddot{a}(0,tR,Ps*SIN(t),t)'$$

EQ15 ' $\phi_{\text{fringes}}=2 \cdot$
 $V \cdot \sin(\hat{i}) / \hat{u}$ '

EQ16 ' $t_R = t + \phi t$ '

EQ17 ' $f = \text{CONST}(c) /$
 \hat{u} '

EQ18 ' $SN = \ddot{o} \cdot Pr / ($
 $\text{CONST}(h) \cdot f)$ '

EQ19 ' $\text{axialmode} = 2$
 $\cdot \phi / (2 \cdot L / \text{CONST}(c))$ '

EQ20 ' $I_{mt} = (I_1 \cdot \sin$
 $(N_1 \cdot w_1 \cdot (t_1 / a_1) + \ddot{o} 1))$
 $^2 / (I_2 \cdot \sin(N_2 \cdot w_2 \cdot$
 $t_1 / a_2) + \ddot{o} 2)) ^2$ '

EQ21 ' $\text{distance} =$
 $\text{CONST}(c) \cdot (\text{Time} / 2)$ '

EQ22 ' $\phi_{VE} / \phi_{VH} = f \cdot$
 t_f '

EQ23 ' $\phi_{VE} = \text{CONST}(c$
 $) \cdot t_f / (2 \cdot TP)$ '

EQ24 ' $\phi_{VH} = \text{CONST}(c$
 $) / (2 \cdot f \cdot TP)$ '

EQ25 ' $\phi_{fH} = 1 / TP$ '

EQ26 ' $B = 1 / t_f$ '

EQ27 ' $\phi_{RE} = \text{CONST}(c$

$$)*t_f/2'$$

EQ28
$$\phi_{RH} = \text{CONST}(c$$

$$)/(2*B)'$$

EQ29
$$PT = Po(\text{rect}*$$

$$((t_1 - t/2)/t) + \text{rect}*($$

$$(t_1 - T + t/2)/t))'$$

EQ30
$$R = \text{CONST}(c)*$$

$$(Time/2)'$$

EQ31
$$\phi_R = \hat{u}/(2*(B/$$

$$f))'$$

EQ32
$$f_{di} = 2/\hat{u}*V_i'$$

EQ33
$$i_{Vi} =$$

$$i_{\Psi} \text{CROSS} i_{ri} * \text{DOT} i_{rR}'$$

EQ34
$$\epsilon_{fdi} = 2*\Psi*$$

$$\phi_{Xi}/\hat{u}'$$

EQ35
$$\phi_{Xi} = \hat{u}*\epsilon_{fdi}/$$

$$(2*\Psi)'$$

EQ36
$$\epsilon_{fd} = 1/T_m'$$

EQ37
$$\phi_X = \hat{u}/(2*\Psi*$$

$$T_m)'$$

EQ38
$$\phi_{X1} = \hat{u}/(2*\phi_0$$

$$)'$$

EQ39
$$\phi_{X2} = \hat{u}/2*(\epsilon*$$

$$f/\Psi)'$$

EQ40 'ϕR1=CONST(c
)*tP/2'

EQ41 'fdi=2/û*
i%CROSSlfiDOT(lR/
absiR)'

EQ42 'ϕX3=û/2*(f/
(%*ri))'

EQ43 'ϕX4=û/2*(f/
ϕò)'

EQ44 'ϕR2=CONST(c
)/(2*B)'

EQ45 'ϕR3=û/(2*(B
/f))'

distance '
distance=CONST(c)*(
Time/2)'

axialmode
672732702.61

modeSPACING 'w(
angular)=2*ϕ/(2*L/
CONST(c))'

TWOBEAMwaveform '
Imt=(I1*SIN(N1*w1*(
t1/a1)+ò1))^2/(I2*

$$\partial^2 \eta)^2$$

INTERNALwaveform

```
'I=SIN(N*w*(t/2))^2
```

$$/ \sin(w*(t/2))^2'$$

IMG

```
GROB 131 64 000000000000000000000000000000000000000000  
0000000000000000000000000000000000000000000000000  
0000000000000000000000000000000000000000000000000  
00000000000000000000000000000081000000000000000000  
0000000000004000000000000000CF000000000000A100000000  
00000068100000000000004600000000000000203000000000008  
81000000000870000603100000000000006000000000842000C81100  
0000000000008100000000087700089C3000000000000E000000008  
02000C97900000000000000003000000080A000000F0000000000000  
00C000000000E00000000000000000000000030000000000000000  
00000000000000C800008FFFF9FFFFFFFFFFFFFFFFFFFFFFFB0000800  
001000000000000000000000CF0000800001000000000000000000  
CF380000800001000000000000000000CF3008000080000100000000  
0000000CF30000800008FFFF9FFFFF300000000CF300000080000000  
0000000CFFFFFFFFFFFF000000800000000000000000CF30000FFFF  
FF80000000000000000000CF30000000000080000000000000CF30  
0000000000000800000000000000CF3000000000000008B000000000  
000CF30000000000000000C7E000000000CF30000000000000000E3
```



```

DETECTOR axialmode
modeSPACING
TWOBEMwaveform
INTERNALwaveform )

RESOLUTION

DIR

fnew 'fnew(Hz
)=2*V/û+f'

çf 'çf(Hz)=2*
V/û'

FREQ 'f(Hz)=
CONST(c)/û'

EQ ( EQ1 EQ2
EQ3 EQ4 EQ5 EQ6 EQ7
EQ8 EQ9 EQ10 EQ11
EQ12 EQ13 EQ14 EQ15
EQ16 EQ17 )

frequency 'f(
Hz)=CONST(c)/û'

time

.00000386

clear

½ { c Time
CST B f t T V f û }

```


PURGE

7

distance 'd(m
)=CONST(c)*(time/2)
,

waveformDOUBLESquare

'Pt=Po(rect*((t1-t/
2)/t)+rect*((t1-T+t
/2)/t))'

HvsEimprovementFACTOR

' $\phi_{VE}/\phi_{VH}=f*t$ '

Evelresolution

' $\phi_{VE}=CONST(c)*t/(2*
T)$ '

HvelresOFwholePULSE

' $\phi_{VH}=CONST(c)/(2*f*
T)$ '

HfrequencyRESOLUTION

' $\phi_{fH}=1/T$ '

Hbandwidth 'B

=1/t'

Erange ' $\phi_{RE}=$

CONST(c)*t/2'

Hrange ' $\phi_{RH}=$

```

CONST(c)/(2*B)'
END
DOPPLER
DIR
COHTIME '2*ç/
çÛ'
çÛ 300000000
COHERENCElength
'2*ç*CONST(c)/çÛ'
BOUNDARYresonance
'û=2*L/n'
RESONANCEplot
'R=SIN(2*ç*X/û)'
X .006825
EQ 'Y=SIN(2*ç
*X/û)'
PPAR {
(-.0065,-1.2995626974)
(.0065,.999663613388)
X 0 (0,0) FUNCTION
Y }
PERIOD 'P=1/f
,
CST {

```

```

DOPPLERequation
DIFFRACTION PERIOD
BOUNDARYresonance
RESONANCEplot )
    DOPPLERequation
    'cf(Hz)=2*V/CONST(c
    )'
    DIFFRACTION '
    I=(SIN(φ*W/û)*SIN(ð
    )/(φ*W/û*SIN(ð)))^2
    ,
    END
    DETECTOR
    DIR
    CST { SIGNAL
    DETECTORcurrent )
    SIGNAL 'ðst=
    SIN(1/(fo-fb))'
    DETECTORcurrent
    'it=ö*CONST(q)/(
    CONST(h)*fo)*(Plo+
    Pst+2*(Plo*Pst)^(1/
    2)*COS((fo-fb)*t+

```

òst))'

END

END

9.1.2.2 LP-140 parameter description

%HP: T(1)A(D)F(.);

DIR

Mpar Library Data

EQ { EQ1 EQ2 EQ3

EQ4 EQ5 }

STARTsolver

½ IMG iLCD 7

FREEZE HALT IMGEQ1

iLCD 7 FREEZE HALT

IMGEQ3 iLCD 7

FREEZE HALT IMGEQ5

iLCD 7 FREEZE HALT

{ EQ1 EQ2 EQ3 EQ4

EQ5 } 'EQ' STO

MINIT MSOLVR

¶

DEFINITIONS

"û = wavelength in meters, '(10.6E-6)' Pr = received
power Watts ' Pt=transmitted' power in Watts Æ = target
reflectance % Ar = target area m ^ 2 Ac = receiver

clear aperture m^2 $\%t$ = transmitted solid beam angle
 SR $\%r$ = received beam solid angle in SR 'T=transmit-
 tance' of optical path '(atmosphere*optical ϵ 1)'
 'R=range' in meters 'no= num 'photns' transmitted'
 'nt= num ' photons' received' 'Er= energy 'received'
 J' 't=time' in seconds 'Et=energy' transmitted J
 'h=Plancks ' 'c = speed 'm'"

CLEANvars

$\frac{1}{2}$ ($\%t$ EQ T R
 $\%r$ $\%T$ Ac Ar ϵ Pt Pr
 no nt Er Et h \hat{u} c t
 Mpar PPAR) PURGE

$\overline{1}$

EQ1 'Pr=Pt/(R^2*(
 $\%t*.0349065))*(\epsilon*Ar$
 $)*(Ac/(R^2*(\%r*$
 $.0349065)))*T^2'$

EQ2 'Pt=Et/t'

EQ3 'Er=Pr*t'

EQ4 'nt=Et/(CONST
 (h)*CONST(c)/ \hat{u})'

EQ5 'no=Er/(CONST
 (h)*CONST(c)/ \hat{u})'

IMGEQ1

```

GROB 268 56 END

9.1.2.3 Linwidth model

[ 6 ]

%%HP: T(1)A(D)F(.);

DIR

  LWTH

    ½ PIC USET ( f0
AXIAL GAS DOPPLER
COLLISION GAIN )
'EQ' STO MINIT
"LINEWIDTH" ( û J
"" æ0 "" L CO2 N2
He "" P T v0 Çvax
Çvd Çvc v æ KPROP )
MITM MSOLVR

  7

  PLT

    ½ 4 ENG GAIN

'EQ' STO DEPTH 0 ==
'SETUP'

    ½ DUP TYPE 12

i 'SETUP'

    ½ DUP OBJ1

SWAP DROP "Çvc" i

```

```

'SETUP' IFT
    7 IFTE
    7 IFTE DRAW
Cvax 'Cvax' iTAG
Cvd 'Cvd' iTAG Cvc
'Cvc' iTAG STD
    7
LIMP
    ½ 0 DUP DUP2 4
DUPN i ŷ Ö L E Cv
FO K R
    ½ 8.314 'R'
STO 16.97 'Cv' STO
.0002 'K' STO
"Pulse energy " ""
INPUT OBJi 'E' STO
"Pulse length" ""
INPUT OBJi 'Ö' STO
"Cavity length" ""
INPUT OBJi 'L' STO
"Spot size" ""
INPUT OBJi 'ŷ' STO
"9*m or 10*m" ""
INPUT OBJi 10 <

```

```

3.87E13 4.17E13
IFTE 'FO' STO '2*K*
FO*E*R*T^3/(3*Ç*ÿ^4
*L*Cv*Ö)' STEQ
ERASE 0 'Ö' RCL
XRNG 'T' INDEP AUTO
DRAX DRAW GRAPH

```

¶

¶

USET

```

½ '20.66_MHz*K^
.5/torr' 'KPROP'
STO '0_m' 'û' STO
'0_MHz' DUP DUP2
DUP 'v' STO 'Çvd'
STO 'Çvc' STO
'Çvax' STO 'v0' STO
'0_m' 'L' STO '0_
torr' 'P' STO '0_K'
'T' STO

```

¶

MESG

"Results for a 1:1:3 CO2:N2:He gas mixture at pressures of 50, 300 and 760 torr and a cavity length of 1.44 m."


```

KPROP '20.66_MHz*
K^.5/torr'

SETUP

  ½  $\phi_{vax}$  UVAL 2 *
1 SCALE v0 UVAL æ0
2 / R1C CENTR v0
UVAL 0 R1C
"Frequency" "Gain"
3 iLIST AXES ERASE
DRAX LABEL

  ¶
PIC

  ½ CAV iLCD 7
FREEZE

  ¶
f0 'v0=CONST(c)/û
'

AXIAL ' $\phi_{vax}$ =CONST
(c)/(2*L)'

GAS 'He=1-CO2-N2'

DOPPLER ' $\phi_{vd}=\hat{a}(8*
LN(2)*CONST(k)*T/(
MOLWT(CO2)/CONST(NA
)*CONST(c)^2))*v0'$ 

```

COLLISION ' $\zeta_{vc}=P*$
 $\hat{a}(KPROP^2/T)*(7.52-$
 $.059*J)*(CO2+.73*N2$
 $+.64*He)'$

GAIN ' $\alpha=\alpha_0*(\zeta_{vc}+$
 $\zeta_{vd})^2/(4*(v-v_0)^2+$
 $(\zeta_{vc}+\zeta_{vd})^2)'$

CAV

GROB 131 64 00000000000000080000000000000000000000000000
000080000000000000000000000000000000000000008000000000000000
000000000000000800000000000000000000000004000000000083000000
0000010000000060000000000000000000000000300000000FFFFFFFF
FFFFFFFFFFFFFFFF7000000006000000000000000000000000003000000
004CFFFFFFFFFFFFFFFFFFFFFFFFF11000000008FFFFFFFFFFFFFFFFFFFFF
FFF00000000008FFFFFFFFFFFFFFFFFFFFFFFFFFFF000000000000FFFFFFFF
FFFFFFFFFFFFFFFF70000000000000EFFFFFFFFFFFFFFFFFFFFFFFF3000000000
0108FFFFFFFFFFFFFFFFFFFFFFFFF00400000002100EFFFFFFFFFFFFFFFFFFFF
30042000000410000000000000000000000000410000008100EFFFFFFF
FFFFFFFFFFFF300C0000000210020000000000000000000200420000004
10020000000000000000000020041000000810020000000000000000002
00C0000000210020000000000000000000200420000004100200000000
00000000002004100000081002000000000000000000200C8F7000021
00208D1428A383830000020042080000410020441C28A082010000020
041001000810020441C38B3830100000200C0202000210020441438A0


```

Data PTpar
Library Data

PPAR (
(28600391.9818,-1.5)
(28603175.769,4.8)
v 0 (
(28601783.8754,0)
"Frequency" "Gain"
) FUNCTION Y )
EQ 'x=x0*(Cvc+Cvd
)^2/(4*(v-v0)^2+(
Cvc+Cvd)^2)'

CST ( LWTH PLT
LIMP )

v '0_MHz'

Cvd '
54.3694151539_MHz'

Cvc '
321.027142182_MHz'

Cvax '
107.068734953_MHz'

v0 '28601783.8754
_MHz'

T '310_K'

```

```

P '60_torr'

He .2

N2 .5

CO2 .3

L '1.4_m'

æ0 3.2

J 30

û '10.4816_*m'

END

9.1.2.4 CO2 description

%%HP: T(1)A(D)F(.);

DIR

    LINEWIDTH {
LINEWIDTH AT 50
TORR IS 300 MHZ OR
4 GHZ AT ATMOSPHERE
    }

    CHIRP ( CHIRP IS
PRESSURE DEPENDENT
LONGER PULSES
PRODUCED BY ADDING
N2 IE LOWER N2 IS
LOWER CHIRP )

    LIMP ( LIMP IS

```

LASER ENERGY
 DEPENDENT AND
 SLIGHTLY ON
 PRESSURE OR BEAM
 SIZE }

 N2 { N2 STORES
 ENERGY AND
 TRANSFERS
 COLLISIONALLY }

 HE { HE COOLS GAS
 BY DEPOPULATING
 MOLECULE INTO
 TRANSLATION ALSO
 REDUCES ARCING }

 CO2 { CO2 CAN
 LASE ALONE BUT WITH
 BROAD LINE AND
 ARCING }

 CWdoppler
 GROB 131 64 PPAR {
 (-6.5,-3.1)
 (6.5,3.2) X 0 (0,0)
 FUNCTION Y }

 END

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